VnK Academy Thane

JEE (Main)-2025 (Online) Session-2

Question Paper with Solutions

(Mathematics, Physics, And Chemistry)

2 April 2025 Shift – 1

Time: 3 hrs. **M.M : 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.

(4) Section - A : Attempt all questions.

(5) Section - B : Attempt all questions.

(6) Section - A (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

(7) Section - B (21 – 25) contains 5 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. The largest $n \in N$ such that 3^n divides 50! is:

(1) 21 (2) 22

(3) 20 (4) 23

Ans. (2)

Sol. $2^{\alpha} \cdot 3^{\beta} \cdot 5^{\gamma}$

$$B = \left[\frac{50}{3}\right] + \left[\frac{50}{3^2}\right] + \left[\frac{50}{3^3}\right] + \left[\frac{50}{3^4}\right]$$

$= 16 + 5 + 1$

$= 2$

Maximum value of n is 22

2. Let one focus of the hyperbola H: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be at $\left(\sqrt{10}, 0\right)$ and the corresponding directrix be $x = \frac{9}{\sqrt{10}}$. If e and l respectively are the eccentricity and the length of the latus rectum of H, then $9(e^2 + l)$ is equal to:

(1) 14 (2) 15

(3) 16 (4) 12

Ans. (3)

Sol. $ae = \sqrt{10}$ and $\frac{a}{e} = \frac{9}{10}$

$\Rightarrow a^2 = 9$ and $e = \frac{\sqrt{10}}{3}$

Now $(ae)^2 = a^2 + b^2$

$10 = 9 + b^2 \Rightarrow b^2 = 1$

$\ell = \frac{2b^2}{a} = \frac{2(1)}{3}$

$\Rightarrow 9(e^2 + \ell)$

$= 9\left(\frac{10}{9} + \frac{2}{3}\right)$

$= 10 + 6$

$= 16$

3. The number of sequences of ten terms, whose terms are either 0 or 1 or 2, that contain exactly five 1s and exactly three 2s, is equal to

(1) 360 (2) 45

(3) 2520 (4) 1820

Ans. (3)

Sol. 11111 2 2 2 0 0

No. of sequences $= \frac{10!}{5!3!2!} = 2520$

Note : Sequence can start with 0.

4. Let $f : \mathbf{R} \to \mathbf{R}$ be a twice differentiable function such that $(\sin x \cos y)(f(2x+2y) - f(2x-2y)) = (\cos x \sin y)(f(2x+2y) + f(2x-2y))$, for all $x, y \in \mathbf{R}$.

If $f'(0) = \frac{1}{2}$, then the value of $24 f''\left(\frac{5\pi}{3}\right)$ is:

(1) 2 (2) –3

(3) 3 (4) –2

Ans. (2)

Sol. $(\sin x \cos y)(f(2x + 2y) - f(2x - 2y)) = (\cos x \sin y)(f(2x + 2y) + f(2x - 2y))$

$f(2x + 2y)(\sin(x-y)) = f(2x - 2y)\sin(x + y)$

$$\frac{f(2x+2y)}{\sin(x+y)} = \frac{f(2x-2y)}{\sin(x-y)}$$

Put $2x + 2y = m$, $2x - 2y = n$

$$\frac{f(m)}{\sin\left(\frac{m}{2}\right)} = \frac{f(n)}{\sin\left(\frac{n}{2}\right)} = K$$

$\Rightarrow f(m) = K \sin\left(\frac{m}{2}\right)$

$\therefore f(x) = K \sin\left(\frac{x}{2}\right)$

$f'(x) = \frac{K}{2}\cos\left(\frac{x}{2}\right)$

Put $x = 0$; $\frac{1}{2} = \frac{K}{2} \Rightarrow K = 1$

$f'(x) = \frac{1}{2}\cos\frac{x}{2}$

$$f''(x) = -\frac{1}{4}\sin\frac{x}{2}$$

$$4f''\left(\frac{5\pi}{3}\right) = \left(-\frac{1}{4}\sin\left(\frac{5\pi}{6}\right)\right)24$$

$$= \frac{-24}{8} = -3$$

5. Let $A = \begin{bmatrix} \alpha & -1 \\ 6 & \beta \end{bmatrix}$, $\alpha > 0$, such that det(A) = 0 and $\alpha + \beta = 1$. If I denotes 2×2 identity matrix, then the matrix $(1 + A)^8$ is:

(1) $\begin{bmatrix} 4 & -1 \\ 6 & -1 \end{bmatrix}$ (2) $\begin{bmatrix} 257 & -64 \\ 514 & -127 \end{bmatrix}$

(3) $\begin{bmatrix} 1025 & -511 \\ 2024 & -1024 \end{bmatrix}$ (4) $\begin{bmatrix} 766 & -255 \\ 1530 & -509 \end{bmatrix}$

Ans. **(4)**

Sol. $|A| = 0$

$\alpha\beta + 6 = 0$

$\alpha\beta = -6$

$\alpha + \beta = 1$

$\Rightarrow \alpha = 3, \beta = -2$

$$A = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}$$

$\therefore A^2 = A$

$A = A^2 = A^3 = A^4 = A^5$

$(I + A)^8$

$= I + {}^8C_1A^7 + {}^8C_2A^6 + \ldots\ldots + {}^8C_8A^8$

$= I + A\,({}^8C_1 + {}^8C_2 + \ldots\ldots + {}^8C_8)$

$= I + A(2^8 - 1)$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 765 & -255 \\ 1530 & -510 \end{bmatrix}$$

$$= \begin{bmatrix} 766 & -255 \\ 1530 & -509 \end{bmatrix}$$

6. The term independent of x in the expansion of $\left(\frac{(x+1)}{\left(x^{2/3}+1-x^{1/3}\right)} - \frac{(x+1)}{\left(x-x^{1/2}\right)}\right)^{10}$, $x > 1$ is:

(1) 210 (2) 150

(3) 240 (4) 120

Ans. **(1)**

Sol. $$\left(\frac{(x+1)}{\left(x^{\frac{2}{3}}+1-x^{\frac{1}{3}}\right)} - \frac{(x-1)}{\left(x-x^{\frac{1}{2}}\right)}\right)^{10}$$

$$= \left(\left(x^{\frac{1}{3}}+1\right) - \left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$$

$$= \left(x^{\frac{1}{3}} - \frac{1}{\sqrt{x}}\right)^{10}$$

$$T_{r+1} = {}^{10}C_r(x)^{\frac{10-r}{3}}(-1)^r(x)^{-\frac{r}{2}}$$

$$\frac{10-r}{3} - \frac{r}{2} = 0$$

$(20 - 2r) - 3r = 0$

$r = 4$

$\Rightarrow {}^{10}C_4(-1)^4 = 210$

7. If $\theta \in [-2\pi, 2\pi]$, then the number of solutions of $2\sqrt{2}\cos^2\theta + \left(2-\sqrt{6}\right)\cos\theta - \sqrt{3} = 0$, is equal to:

(1) 12 (2) 6

(3) 8 (4) 10

Ans. **(3)**

Sol. $2\sqrt{2}\cos^2\theta + 2\cos\theta - \sqrt{6}\cos\theta - \sqrt{3} = 0$

$(2\cos\theta - \sqrt{3})\ (\sqrt{2}\cos\theta + 1) = 0$

$$\cos\theta = \frac{\sqrt{3}}{2}, \frac{-1}{\sqrt{2}}$$

Number of solution = 8

8. Let a_1, a_2, a_3…. be in an A.P. such that $\sum_{k=1}^{12} a_{2k-1} = -\frac{72}{5}a_1, a_1 \neq 0$. If $\sum_{k=1}^{n} a_k = 0$, then n is:

(1) 11 (2) 10

(3) 18 (4) 17

Ans. (1)

Sol. Let $a_1 = a$, common difference = d

$$a_1 + a_3 + a_5 + \ldots\ldots + a_{23} = -\frac{72}{5}a$$

$$\frac{12}{2}[2a + 11 \times 2d] = -\frac{72}{5}a$$

$$12a + 132d = -\frac{72}{5}a$$

$$132a + 132 \times 5d = 0$$

$$a = -5d$$

$$\frac{n}{2}(2a + (n-1)d) = 0 \Rightarrow -10d + nd - d = 0$$

$$n = 11$$

9. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its local maximum and local minimum values at p and q, respectively, such that $p^2 = q$, then $f(3)$ is equal to:

(1) 55 (2) 10

(3) 23 (4) 37

Ans. (4)

Sol. $f'(x) = 6x^2 - 18ax + 12a^2$

$f'(x) = 6(x^2 - 3ax + 2a^2)$

roots are a, 2a

$p^2 = q \Rightarrow a^2 = 2a$

$a = 2$

$f(x) = 2x^3 - 18x^2 + 48x + 1$

$f(3) = 37$

10. Let z be a complex number such that $|z| = 1$. If $\frac{2+k^2z}{k+\bar{z}} = kz$, $k \in \mathbf{R}$, then the maximum distance of $k + ik^2$ from the circle $|z - (1 + 2i)| = 1$ is:

(1) $\sqrt{5}+1$ (2) 2

(3) 3 (4) $\sqrt{3}+1$

Ans. (1)

Sol. $\frac{2+k^2z}{k+\bar{z}} = kz$

$|z|^2 k = 2$

$k = 2$

point p(2, 4); center (1, 2)

distance from circle

$(x - 1)^2 + (y - 2)^2 = 1$ is max.

if $(OP + r) = \sqrt{1+4}+1 = \sqrt{5}+1$

11. If $\vec{a}$ is nonzero vector such that its projections on the vectors $2\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} + 2\hat{j} - 2\hat{k}$ and $\hat{k}$ are equal, then a unit vector along $\vec{a}$ is:

(1) $\frac{1}{\sqrt{155}}\left(-7\hat{i}+9\hat{j}+5\hat{k}\right)$ (2) $\frac{1}{\sqrt{155}}\left(-7\hat{i}+9\hat{j}-5\hat{k}\right)$

(3) $\frac{1}{\sqrt{155}}\left(7\hat{i}+9\hat{j}+5\hat{k}\right)$ (4) $\frac{1}{\sqrt{155}}\left(7\hat{i}+9\hat{j}-5\hat{k}\right)$

Ans. (3)

Sol. Let $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$$a_1^2 + a_2^2 + a_3^2 = 1$$

Let $\bar{b} = 2\vec{i} - \hat{j} + 2\hat{k}$, $\bar{c} = \vec{i} - 2\hat{j} - 2\hat{k}$

$\bar{d} = \hat{k}$

$$\frac{\bar{a}.\bar{b}}{|b|} = \frac{\bar{a}.\bar{c}}{|c|} = \frac{\bar{a}.\bar{d}}{|d|}$$

$$\frac{2a_1 - a_2 + 2a_3}{3} = \frac{a_1 + 2a_2 - 2a_3}{3} = a_3$$

By solving

$$a_1 = \frac{7}{\sqrt{155}}, a_2 = \frac{9}{\sqrt{155}}, a_3 = \frac{5}{\sqrt{155}}$$

12. Let A be the set of all functions $f: \mathbf{Z} \rightarrow \mathbf{Z}$ and R be a relation on A such that R = {(f, g) : $f(0) = g(1)$ and $f(1) = g(0)$}. Then R is:

(1) Symmetric and transitive but not reflective

(2) Symmetric but neither reflective nor transitive

(3) Reflexive but neither symmetric nor transitive

(4) Transitive but neither reflexive nor symmetric

Ans. (2)

Sol. R = {(f, g) : f(0) = g(1) and f(1) = g(0)}

Reflexive: (f, f) $\in$ R

= f(0) = f(1) and f(1) = f(0) $\rightarrow$ must hold

$\Rightarrow$ but this is not true for all function

so not reflexive

Symmetric: If (f, g) $\in$ R $\Rightarrow$ (g, f) $\in$ R

Now, g(0) = f(1) and g(1) = f(0) $\rightarrow$ true

$\therefore$ symmetric

Transitive : If(f, g) $\in$ R and (g, h) $\in$ R

$\Rightarrow$ (f, h) $\in$ R

Now (f, g) $\in$ R $\Rightarrow$ f(0) = g(1) and f(1) = g(0)

(g, h) $\in$ R $\Rightarrow$ g(0) = h(1) and g(1) = h(0)

For (f, h) $\in$ R we need f(0) = h(1) and f(1) = h(0)

Now f(0) = g(1) = h(0) and f(1) = g(0) = h(1)

Hence not transitive

13. For $\alpha, \beta, \gamma, \in \mathbf{R}$, if $\lim\limits_{x\to 0}\dfrac{x^2\sin\alpha x+(\gamma-1)e^{x^2}}{\sin 2x-\beta x}=3$, then $\beta+\gamma-\alpha$ is equal to:

(1) 7 (2) 4

(3) 6 (4) –1

Ans. (1)

Sol. $$\lim_{x\to 10}\frac{x^2(\alpha x)+(\gamma-1)\left(1+\frac{x^2}{1}\right)}{2x-\frac{8x^3}{6}-\beta x}=3$$

$$\lim_{x\to 0}\frac{(\gamma-1)+(\gamma-1)x^2+\alpha x^3}{(2-\beta)x-\frac{4}{3}x^3}=3$$

$$\gamma-1,\ \beta=2,\ \frac{-3\alpha}{4}=+3\ \Rightarrow \alpha=-4$$

$$\beta+\gamma-\alpha=7$$

14. If the system of linear equations

$3x + y + \beta z = 3$

$2x + \alpha y - z = -3$

$x + 2y + z = 4$

has infinitely many solutions, then the value of $22\beta - 9\alpha$ is :

(1) 49 (2) 31

(3) 43 (4) 37

Ans. (2)

Sol. $$\Delta=\begin{vmatrix}3 & 1 & \beta\\ 2 & \alpha & -1\\ 1 & 2 & 1\end{vmatrix}=0$$

$$3\alpha+4\beta-\alpha\beta+3=0$$

$$\Delta_3=\begin{vmatrix}3 & 1 & 3\\ 2 & \alpha & -3\\ 1 & 2 & 4\end{vmatrix}=0$$

$$9\alpha+19=0$$

$$\alpha=\frac{-19}{9},\ \beta=\frac{6}{11}$$

$$\Rightarrow 22\beta-9\alpha=31$$

15. Let $P_n = \alpha^n + \beta^n$, $n \in \mathbf{N}$. If $P_{10} = 123$, $P_9 = 76$, $P_8 = 47$ and $P_1 = 1$, then the quadratic equation having roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is :

(1) $x^2 - x + 1 = 0$ (2) $x^2 + x - 1 = 0$

(3) $x^2 - x - 1 = 0$ (4) $x^2 + x + 1 = 0$

Ans. (2)

Sol. $\alpha^{10} + \beta^{10} = 123$

$\alpha + \beta = 1$

$\alpha^9 + \beta^9 = 76$

$\alpha^8 + \beta^8 = 47$

$P_{10} = P_9 + P_8$

$x^2 = x + 1 \Rightarrow x^2 - x - 1 = 0$

$\alpha + \beta = 1,\ \alpha\beta = -1$

$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{1}{-1}=-1,\ \frac{1}{\alpha\beta}=-1$$

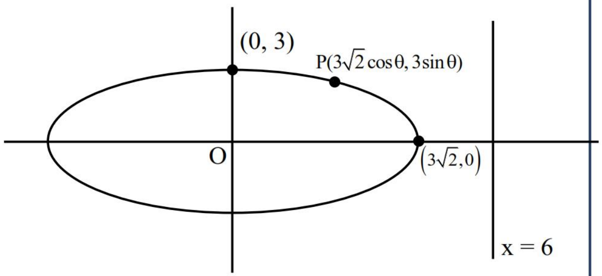
16. If S and S' are the foci of the ellipse $\frac{x^2}{18}+\frac{y^2}{9}=1$ and P be a point on the ellipse, then min(SP.S'P) + max(SP.S'P) is equal to :

(1) $3\left(1+\sqrt{2}\right)$ (2) $3\left(6+\sqrt{2}\right)$

(3) 9 (4) 27

Ans. (4)

Sol.



$PS + PS' = 2 \times 3\sqrt{2}$

$b^2 = a^2(1-e^2) \Rightarrow 9 = 18(1-e^2)$

$\Rightarrow e = \dfrac{1}{\sqrt{2}}$

Directrix $x = \dfrac{a}{e} = \dfrac{3\sqrt{2}}{\frac{1}{\sqrt{2}}} = 6$

$PS \cdot PS' = \left|\dfrac{1}{\sqrt{2}}(3\sqrt{2}\cos\theta - 6)\dfrac{1}{\sqrt{2}}(3\sqrt{2}\cos\theta + 6)\right|$

$= \dfrac{1}{2}\left|18\cos^2\theta - 36\right|$

$(PS \cdot PS')_{max} = 18$; $(PS \cdot PS')_{min} = 9$

sum = 27

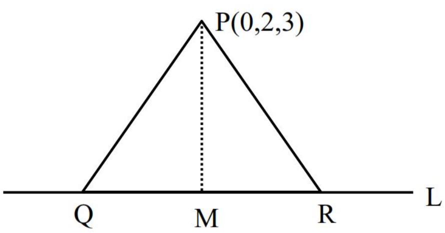
17. Let the vertices Q and R of the triangle PQR lie on the line $\dfrac{x+3}{5} = \dfrac{y-1}{2} = \dfrac{z+4}{3}$, QR = 5 and the coordinates of the point P be (0, 2, 3). If the area of the triangle PQR is $\dfrac{m}{n}$ then :

(1) $m - 5\sqrt{21}\,n = 0$ (2) $2m - 5\sqrt{21}\,n = 0$

(3) $5m - 2\sqrt{21}\,n = 0$ (4) $5m - 21\sqrt{2}\,n = 0$

Ans. **(2)**

Sol.



QR = 5

$M(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$

Drs of PM $\Rightarrow 5\lambda - 3, 2\lambda - 1, 3\lambda - 7$

Drs of line L $\Rightarrow$ 5, 2, 3

PM $\perp$ L

$\Rightarrow (5\lambda - 3)5 + (2\lambda - 1)2 + (3\lambda - 7)3 = 0$

$\Rightarrow \lambda = 1$

$\therefore M(2, 3, -1)$

$PM = \sqrt{4+1+16} = \sqrt{21}$

Area $= \dfrac{1}{2} \times 5 \times \sqrt{21} = \dfrac{m}{n}$

$2m - 5\sqrt{21}\,n = 0$

18. Let ABCD be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the areas of the triangles ABC, ACD and ADB be 5, 6 and 7 square units respectively. Then the area (in square units) of the ΔBCD is equal to :

(1) $\sqrt{340}$ (2) 12

(3) $\sqrt{110}$ (4) $7\sqrt{3}$

Ans. **(3)**

Sol. $Ar(\Delta BCD)$

$= \sqrt{(Ar(\Delta ABC))^2 + (Ar(ACD))^2 + (Ar(\Delta ADB))^2}$

$= \sqrt{5^2 + 6^2 + 7^2}$

$= \sqrt{110}$

19. Let $a \in \mathbf{R}$ and A be a matrix of order 3×3 such that $\det(A) = -4$ and $A + I = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix}$, where I is the identity matrix of order 3×3.

If det ((a + 1)adj((a–1)A)) is $2^m 3^n$, m, n $\in$ {0,1,2,…..20}, then m + n is equal to :

(1) 14 (2) 17

(3) 15 (4) 16

Ans. **(4)**

Sol. $A = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix} - I = \begin{bmatrix} 0 & a & 1 \\ 2 & 0 & 0 \\ a & 1 & 1 \end{bmatrix}$

$|A| = -4 \Rightarrow 2 - 2a = -4 \Rightarrow a = 3$

$|(a + 1)\ \text{adj}\ (a - 1)A| = |4\ \text{adj}\ 3A|$

$= 4^3|\text{adj}\ 3A|$

$= 4^3 \times |3A|^{3-1} = 64|3A|^2$

$= 64 \times (3^3)^2\ |A|^2$

$= 2^6 \times 3^6 \times 16$

$2^m \times 3^n = 2^{10} \times 3^6$

$\therefore\ m = 10, n = 6$

$\Rightarrow\ m + n = 16$

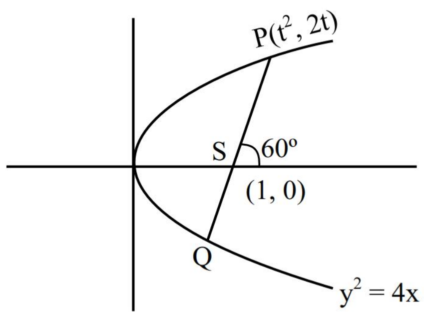
20. Let the focal chord PQ of the parabola $y^2 = 4x$ make an angle of 60° with the positive x-axis, where P lies in the first quadrant. If the circle, whose one diameter is PS, S being the focus of the parabola, touches the y-axis at the point $(0, \alpha)$, then $5\alpha^2$ is equal to :

(1) 15 (2) 25

(3) 30 (4) 20

Ans. **(1)**

Sol.



$\tan 60° = \dfrac{2t - 0}{t^2 - 1} = \sqrt{3} \quad \Rightarrow t = \sqrt{3}$

$\therefore\ P(3, 2\sqrt{3})$

Circle :

$(x - 1)(x - 3) + (y - 0)(y - 2\sqrt{3}) = 0$

at $x = 0$

$\Rightarrow 3 + y^2 - 2\sqrt{3}\,y = 0$

$\Rightarrow y = \sqrt{3} = \alpha$

$5\alpha^2 = 15$

SECTION-B

21. Let $[\cdot]$ denote the greatest integer function. If $\int_0^{e^3}\left[\dfrac{1}{e^{x-1}}\right]dx = \alpha - \log_e 2$, then α^3 is equal to _____.

Ans. **(8)**

Sol. $f(x) = \dfrac{1}{e^{x-1}} = e^{1-x}$

$$\begin{array}{l|l} f(x) = 2 & f(x) = 1 \\ \dfrac{1}{e^{x-1}} = 2 & x = 1 \\ x = 1 - \ell n2 & \end{array}$$

$f(0) = e^1 = 2.71$

$f(e^3) = e^{1-e^3} \in (0,1)$

$I = \int_0^{1-\ell n2} 2dx + \int_{1-\ell n2}^{1} 1dx + \int_1^{e^3} 0dx$

$= 2(1 - \ell n2 - 0) + 1(1 - 1 + \ell n2) + 0$

$\alpha - \ell n2 = 2 - \ell n2$

$\alpha = 2$

$\alpha^3 = 8$

22. Let f : **R→R** be a thrice differentiable odd function satisfying

$f'(x) \geq 0,\ f''(x) = f(x), f(0) = 0, f'(0) = 3$. Then $9f(\log_e 3)$ is equal to _____.

Ans. **(36)**

Sol. $f''(x) = f(x)$

$\Rightarrow f'(x)\,.\,f''(x) = f'(x)\cdot f(x)$

$\Rightarrow \dfrac{(f'(x))^2}{2} = \dfrac{(f(x))^2}{2} + C$

$\Rightarrow (f'(x))^2 = (f(x))^2 + C'$

$f(0) = 0,\ f'(0) = 3 \qquad \Rightarrow C' = 9$

$\therefore\ (f'(x))^2 = (f(x))^2 + 9$

$f'(x) = \sqrt{(f(x))^2 + 9} \qquad \because\ f'(x) \geq 0$

$\int \dfrac{dy}{\sqrt{y^2 + 9}} = \int dx \Rightarrow \ell n\left|y + \sqrt{y^2 + 9}\right| = x + C$

$\Rightarrow f(0) = 0 \Rightarrow C = \ell n3$

$\Rightarrow\ y + \sqrt{y^2 + 9} = 3e^x$

at $x = \ell n3$; $y = 4$

$\therefore\ 9f(\ell n3) = 36$

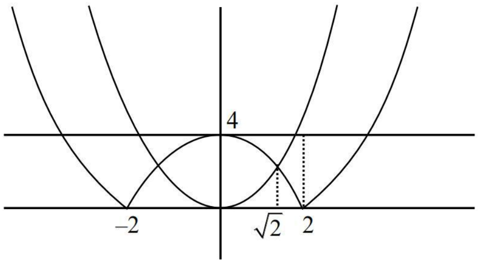
23. If the area of the region

$$\{(x,y):|4-x^2|\le y\le x^2, y\le 4, x\ge 0\}$$

is $\left(\frac{80\sqrt{2}}{\alpha}-\beta\right)$, **α, β ∈ N**, then $\alpha+\beta$ is equal to _____.

Ans. (22)

Sol.



$$A=\int_0^4\sqrt{4+y}\,dy-\int_0^2\sqrt{4-y}\,dy-\int_2^4\sqrt{y}\,dy$$

$$=\left(\frac{(4+y)^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^4+\left(\frac{(4-y)^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^2-\left(\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^4$$

$$\frac{80\sqrt{2}}{3}-16=\frac{40\sqrt{2}}{3}-16$$

$\alpha=6,\ \beta=16$

$\alpha+\beta=22$

24. Three distinct numbers are selected randomly from the set $\{1,2,3,\ldots\ldots,40\}$. If the probability, that the selected numbers are in an increasing G.P. is $\frac{m}{n}$, $\gcd(m,n)=1$, then $m+n$ is equal to ____.

Ans. (4949)

Sol. $1\le a<ar<ar^2\le 40$

(If $r\in N$)

<u>If r = 2</u>

$1\le a<2a<4a\le 40$

$a\in\{1,\ldots\ldots,10\}$ (10 GP)

<u>If r = 3</u>

$1\le a<3a<9a\le 40$

$a\in\{1,2,3,4\}$ (4 GP)

<u>If r = 4</u>

$1\le a<4a<16a\le 40$

$a\in\{1,2\}$ (2 GP)

<u>If r = 5</u>

$1\le a<5a<25a\le 40$

$a\in\{1\}$ (1 GP)

<u>If r = 6</u>

$1\le a<6a<36a\le 40$

$a\in\{1\}$ (1 GP)

$\left(P=\frac{18}{9880}=\frac{9}{4940}\right)$ as per NTA for $r\in N$

$m+n=4949$

If <u>$r\notin N$</u> (also possible)

$r=\frac{3}{2}$

$ar^2=\frac{9a}{4}$; $a=4k$

$$\left.\begin{matrix}(4,6,9)\\(8,12,18)\\(12,18,27)\\(16,24,36)\end{matrix}\right\}4GP$$

$r=\frac{5}{2}$ $\quad ar^2=\frac{25a}{4}$; $a=4k$

(4, 10, 25) ....(1) GP

$r=\frac{4}{3}$ $\quad ar^2=\frac{16a}{9}\rightarrow a=9k$

(9, 12, 16), (18, 24, 32) ....(2) GP

$r=\frac{5}{3}$ $\quad ar^2=\frac{25a}{9}$; $a=9k$

(9, 15, 25) ....(1) GP

$r=\frac{5}{4}$ $\quad ar^2=\frac{25a}{16}$; $a=16k$

(16, 20, 25) ....(1) GP

$r=\frac{6}{5}$ $\quad ar^2=\frac{36a}{25}$; $a=25k$

(25, 30, 36) ....(1) GP

Total = 18 + 10 = 28

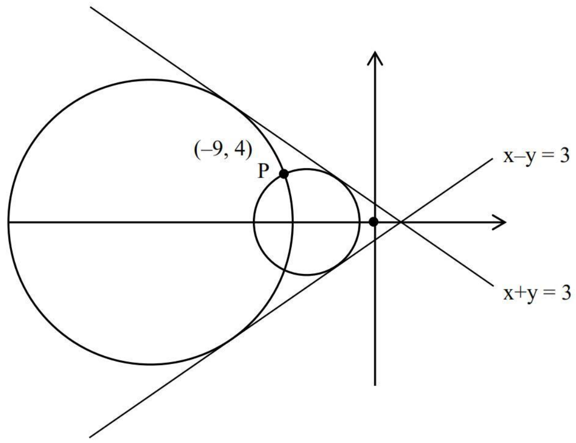
$$P=\frac{28}{{}^{40}C_3}=\frac{28}{9880}=\frac{7}{2470}$$

$m+n=2477$

25. The absolute difference between the squares of the radii of the two circles passing through the point $(-9, 4)$ and touching the lines $x + y = 3$ and $x - y = 3$, is equal to _____.

Ans. (768)

Sol.



Centre $(a, 0)$

$$r = \left|\frac{a-0-3}{\sqrt{2}}\right|$$

$$\text{circle } (x-a)^2 + y^2 = \left(\frac{a-3}{\sqrt{2}}\right)^2$$

passes through $(-9, 4)$

$2(a^2 + 18a + 81 + 16) = (a^2 - 6a + 9)$

$a^2 + 42a + 185 = 0$

$(a + 37)(a + 5) = 0$

$\Rightarrow a = -37, -5$

$$r_1 = \left|\frac{-37-3}{\sqrt{2}}\right| = 20\sqrt{2}$$

$$r_2 = \left|\frac{-5-3}{\sqrt{2}}\right| = 4\sqrt{2}$$

$$\left|r_1^2 - r_2^2\right| = |800 - 32| = 768$$

PHYSICS	TEST PAPER WITH SOLUTION

SECTION-A

26. A light wave is propagating with plane wave fronts of the type x + y + z = constant. The angle made by the direction of wave propagation with the x-axis is :

(1) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (2) $\cos^{-1}\left(\frac{2}{3}\right)$

(3) $\cos^{-1}\left(\frac{1}{3}\right)$ (4) $\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

Ans. (1)

Sol. The direction of propagation of light is perpendicular to the wave front and is symmetric about x, y and z axis.

$\therefore$ Angle made by the light with x, y & z axis is same.

$\therefore$ $\cos\alpha = \cos\beta = \cos\gamma$ (α, β & γ are angle made by light with x, y & z axis respectively)

Also $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ [Sum of direction cosines]

$\therefore\ \alpha = \cos^{-1}\frac{1}{\sqrt{3}}$

27. The equation for real gas is given by $\left(P + \frac{a}{V^2}\right)(V - b) = RT$, where P,V,T and R are the pressure, volume, temperature and gas constant, respectively. The dimension of ab^{-2} is equivalent to that of :

(1) Planck's constant (2) Compressibility

(3) Strain (4) Energy density

Ans. (4)

Sol. $\left[P + \frac{a}{V^2}\right](V - b) = RT$

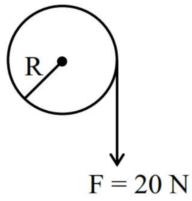
$\therefore$ $[a] = [P]\,[V^2] = ML^{-1}T^{-2}L^6 = ML^5T^{-2}$

$[b] = [V] = L^3$

$[ab^{-2}] = ML^5T^{-2}L^{-6} = ML^{-1}T^{-2}$

Dimension of energy density.

28. A cord of negligible mass is wound around the rim of a wheel supported by spokes with negligible mass. The mass of wheel is 10 kg and radius is 10 cm and it can freely rotate without any friction. Initially the wheel is at rest. If a steady pull of 20 N is applied on the cord, the angular velocity of the wheel, after the cord is unwound by 1 m, would be :



(1) 20 rad/s (2) 30 rad/s

(3) 10 rad/s (4) 0 rad/s

Ans. (1)

Sol. $W_F = 20 \times 1 = 20$ J

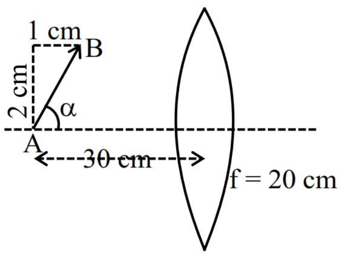
$\therefore\ \Delta KE = 20\text{ J} = \frac{1}{2}I\omega^2$

$I = MR^2 = 10 \times 0.1^2 = 0.1$ kg m^2

$\therefore\ 20 = \frac{1}{2} \times 0.1 \times \omega^2$

$\Rightarrow \omega = 20$ rad/sec

29. A slanted object AB is placed on one side of convex lens as shown in the diagram. The image is formed on the opposite side. Angle made by the image with principal axis is :

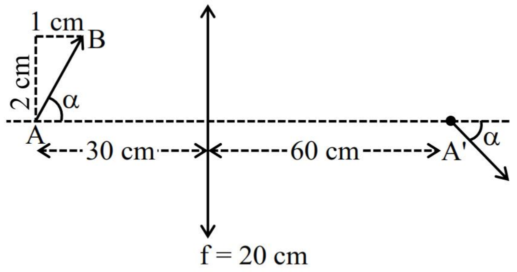


(1) $-\frac{\alpha}{2}$ (2) –45°

(3) +45° (4) –α

Ans. (2)

Sol.



Location of image of A :-

$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}-\frac{1}{-30}=\frac{1}{20} \Rightarrow \frac{1}{v}=\frac{1}{60} \Rightarrow v = 60\text{cm}$

$\therefore$ m = 2

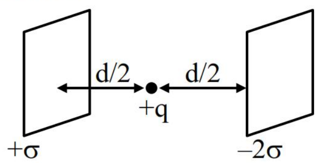
Since size of object is small wrt the location hence

$dv = m^2 du \Rightarrow dv = 4 \times 1 = 4$ cm

$h_i = mh_0 \Rightarrow h_i(dy) = 2 \times 2 = 4$ cm

$\therefore$ Angle made with principle axis = –45°

30. Consider two infinitely large plane parallel conducting plates as shown below. The plates are uniformly charged with a surface charge density $+\sigma$ and -2σ. The force experienced by a point charge +q placed at the mid point between two plates will be :

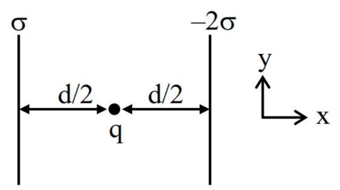


(1) $\frac{\sigma q}{4\epsilon_0}$ (2) $\frac{3\sigma q}{2\epsilon_0}$

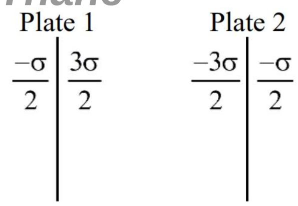
(3) $\frac{3\sigma q}{4\epsilon_0}$ (4) $\frac{\sigma q}{2\epsilon_0}$

Ans. (2)

Sol.



Final charge distribution will be



$\therefore F_{net} = \frac{3\sigma}{2\epsilon_0}q$

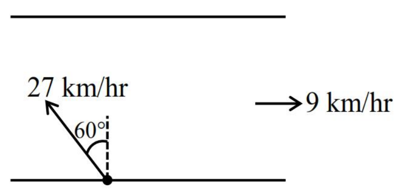
31. A river is flowing from west to east direction with speed of 9 km h^{-1}. If a boat capable of moving at a maximum speed of 27 km h^{-1} in still water, crosses the river in half a minute, while moving with maximum speed at an angle of 150° to direction of river flow, then the width of the river is :

(1) 300 m (2) 112.5 m

(3) 75 m (4) $112.5 \times \sqrt{3}$ m

Ans. (2)

Sol.



$\therefore V_{\perp}$ = river flow $= 27 \times \cos 60° = \frac{27}{2}$ km / hr.

Time taken = 30 sec.

$\therefore S = Vt = \frac{27}{2}\times\frac{5}{18}\times 30\text{m} = 112.5\text{m}$

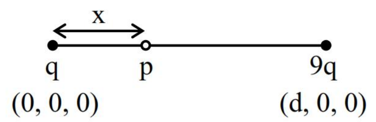
32. A point charge +q is placed at the origin. A second point charge +9q is placed at (d, 0, 0) in Cartesian coordinate system. The point in between them where the electric field vanishes is :

(1) (4d/3, 0, 0) (2) (d/4, 0, 0)

(3) (3d/4, 0, 0) (4) (d/3, 0, 0)

Ans. (2)

Sol.



Let $E_P = 0$

$$\therefore \frac{kq}{x^2} = \frac{k9q}{(d-x)^2}$$

$$\Rightarrow \frac{d-x}{x} = 3 \Rightarrow x = \frac{d}{4}$$

$\therefore$ co-ordinate of P is $\left(\frac{d}{4}, 0, 0\right)$

33. The battery of a mobile phone is rated as 4.2 V, 5800 mAh. How much energy is stored in it when fully charged ?

(1) 43.8 kJ (2) 48.7 kJ

(3) 87.7 kJ (4) 24.4 kJ

Ans. (3)

Sol. Given V = 4.2 volt

$\therefore$ Energy supplied by battery

$= vq = 4.2 \times 5800 \times 3600 \times 10^{-3}$ J = 87.696 kJ

$\therefore$ Energy stored in the battery when fully charged

= 87.696 kJ ≈ 87.7 kJ

34. A particle is subjected two simple harmonic motions as :

$$x_1 = \sqrt{7}\sin 5t \text{ cm}$$

and $x_2 = 2\sqrt{7}\sin\left(5t + \frac{\pi}{3}\right)$ cm

where x is displacement and *t* is time in seconds. The maximum acceleration of the particle is $x \times 10^{-2}$ ms^{-2}. The value of x is :

(1) 175 (2) $25\sqrt{7}$

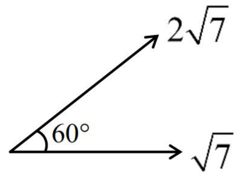
(3) $5\sqrt{7}$ (4) 125

Ans. (1)

Sol. $x_1 = \sqrt{7}\sin 5t$

$\mathbf{x_2} = 2\sqrt{7}\sin\left(5t + \frac{\pi}{3}\right)$

From phasor,



$\therefore$ Amplitude of resultant SHM = 7

$$\phi = \tan^{-1}\frac{2\sqrt{7} \times \sqrt{3}/2}{\sqrt{7} + 2\sqrt{7} \times \frac{1}{2}} = \tan^{-1}\frac{\sqrt{21}}{2\sqrt{7}} = \tan^{-1}\frac{\sqrt{3}}{2}$$

$\therefore$ $X_R = 7\sin(5t + \phi)$

$a_R = -7 \times 25 \sin(5t + \phi)$

$\therefore$ a_{max} = 175 cm/sec = 175×10^{-2} m/sec

35. The relationship between the magnetic susceptibility (χ) and the magnetic permeability (μ) is given by :

(μ_0 is the permeability of free space and μ_r is relative permeability)

(1) $\chi = \frac{\mu}{\mu_0} - 1$ (2) $\chi = \frac{\mu_r}{\mu_0} + 1$

(3) $\chi = \mu_r + 1$ (4) $\chi = 1 - \frac{\mu}{\mu_0}$

Ans. (1)

Sol. We have

$\mu_r = (1 + \chi) \Rightarrow \chi = (\mu_r - 1)$

$\mu = \mu_0\mu_r \Rightarrow \mu_r = \frac{\mu}{\mu_0}$

$$\therefore \chi = \left(\frac{\mu}{\mu_0} - 1\right)$$

36. A zener diode with 5V zener voltage is used to regulate an unregulated dc voltage input of 25 V. For a 400 Ω resistor connected in series, the zener current is found to be 4 times load current. The load current (I_L) and load resistance (R_L) are :

(1) $I_L = 20$ mA; $R_L = 250\ \Omega$

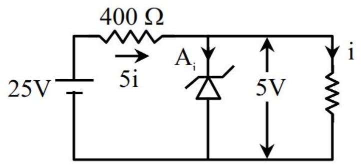
(2) $I_L = 10$ A; $R_L = 0.5\ \Omega$

(3) $I_L = 0.02$ mA; $R_L = 250\ \Omega$

(4) $I_L = 10$ mA; $R_L = 500\ \Omega$

Ans. **(4)**

Sol.



From the circuit diagram,

$$5i = \frac{20}{400} = \frac{1}{20}\text{A}$$

$$\therefore\ i = \frac{1}{100}\text{A} = 10\text{ mA} = \text{Load current}$$

Also, $V_L = 5$ V

$$\therefore\ R_L = \frac{5}{10\times10^{-3}}\Omega = 500\ \Omega$$

37. In an adiabatic process, which of the following statements is true ?

(1) The molar heat capacity is infinite

(2) Work done by the gas equals the increase in internal energy

(3) The molar heat capacity is zero

(4) The internal energy of the gas decreases as the temperature increases

Ans. **(3)**

Sol. For adiabatic process, dQ = 0

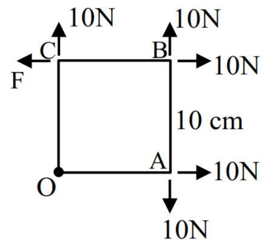
$\therefore$ Molar heat capacity = 0

$\because dQ = 0 \Rightarrow dU = -dW$

Also $dU = \frac{f}{2}nRdT$

$\therefore$ Only option (3) is correct.

38. A square Lamina OABC of length 10 cm is pivoted at 'O'. Forces act at Lamina as shown in figure. If Lamina remains stationary, then the magnitude of F is :



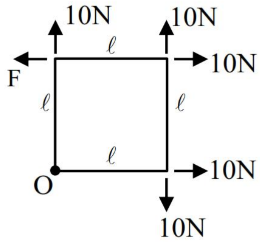
(1) 20 N (2) 0 (zero)

(3) 10 N (4) $10\sqrt{2}$ N

Ans. **(3)**

Sol. Since the lamina is equilibrium.

$\therefore F_{net} = 0\ \&\ \tau_{net} = 0$



$T_O = 10\ell - F\ell \Rightarrow F = 10$ N

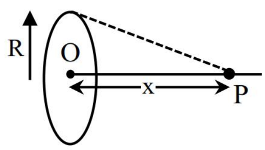
39. Let B_1 be the magnitude of magnetic field at center of a circular coil of radius R carrying current I. Let B_2 be the magnitude of magnetic field at an axial distance 'x' from the center. For x : R = 3 : 4, $\frac{B_2}{B_1}$ is :

(1) 4 : 5 (2) 16 : 25

(3) 64 : 125 (4) 25 : 16

Ans. **(3)**

Sol.



$B_1 = \frac{\mu_0 i}{2R}$ $B_2 = B_1\sin^3\theta$

$$\therefore\ \frac{B_2}{B_1} = \sin^3\theta = \left(\frac{4}{5}\right)^3 = \frac{64}{125}$$

40. Considering Bohr's atomic model for hydrogen atom :

(A) the energy of H atom in ground state is same as energy of He^{+} ion in its first excited state.

(B) the energy of H atom in ground state is same as that for Li^{++} ion in its second excited state.

(C) the energy of H atom in its ground state is same as that of He^{+} ion for its ground state.

(D) the energy of He^{+} ion in its first excited state is same as that for Li^{++} ion in its ground state

Choose the **correct** answer from the options given below :

(1) (B), (D) only (2) (A), (B) only

(3) (A), (D) only (4) (A), (C) only

Ans. (2)

Sol. $E \propto \dfrac{Z}{n^2}$

$Z_H = 1 \qquad Z_{He^+} = 2 \qquad Z_{Li^{+2}} = 3$

1^{st} excited state $\Rightarrow n = 2$

2^{nd} excited state $\Rightarrow n = 3$

From the given statements only A & B are correct.

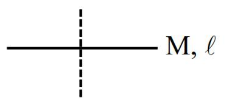
41. Moment of inertia of a rod of mass 'M' and length 'L' about an axis passing through its center and normal to its length is 'α'. Now the rod is cut into two equal parts and these parts are joined symmetrically to form a cross shape. Moment of inertia of cross about an axis passing through its center and normal to plane containing cross is :

(1) α (2) α/4

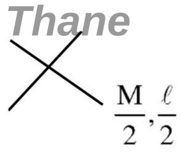
(3) α/8 (4) α/2

Ans. (2)

Sol.



$$\alpha = \frac{M\ell^2}{12} \qquad \ldots \text{(i)}$$

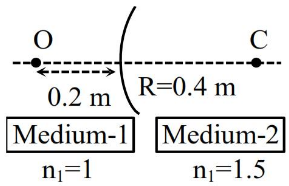


$$\alpha' = 2\left[\frac{\frac{M}{2}\left(\frac{\ell}{2}\right)^2}{12}\right]$$

$$\alpha' = \frac{M\ell^2}{48} = \frac{\alpha}{4}$$

Correct option is (2)

42.



A spherical surface separates two media of refractive indices 1 and 1.5 as shown in figure. Distance of the image of an object 'O', is :

(C is the center of curvature of the spherical surface and R is the radius of curvature)

(1) 0.24 m right to the spherical surface

(2) 0.4 m left to the spherical surface

(3) 0.24 m left to the spherical surface

(4) 0.4 m right to the spherical surface

Ans. (2)

Sol. $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

$$\frac{1.5}{v} - \frac{1}{(-0.2)} = \frac{1.5 - 1}{0.4}$$

$$\frac{1.5}{v} = \frac{0.5}{0.4} - \frac{1}{0.2}$$

$$\frac{1.5}{v} = -\frac{1.5}{0.4}$$

$v = -0.4$ m

43. Match **List–I** with **List–II**.

List–I	**List–II**
(A) Coefficient of viscosity	(I) $[ML^0T^{-3}]$
(B) Intensity of wave	(II) $[ML^{-2}T^{-2}]$
(C) Pressure gradient	(III) $[M^{-1}LT^2]$
(D) Compressibility	(IV) $[ML^{-1}T^{-1}]$

Choose the **correct** answer from the options given below :

(1) (A)–(I), (B)–(IV), (C)–(III), (D)–(II)

(2) (A)–(IV), (B)–(I), (C)–(II), (D)–(III)

(3) (A)–(IV), (B)–(II), (C)–(I), (D)–(III)

(4) (A)–(II), (B)–(III), (C)–(IV), (D)–(I)

Ans. (2)

Sol. (A) Coefficient of viscosity

$[\eta] = [M^1L^{-1}T^{-1}]$

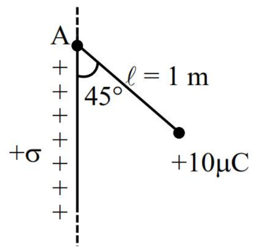
(B) Intensity $[I] = [M^1L^0T^{-3}]$

(C) Pressure gradient $= [ML^{-2}T^{-2}]$

(D) Compressibility $[K] = [M^{-1}L^1T^2]$

44. A small bob of mass 100 mg and charge +10 μC is connected to an insulating string of length 1 m. It is brought near to an infinitely long non–conducting sheet of charge density 'σ' as shown in figure. If string subtends an angle of 45° with the sheet at equilibrium the charge density of sheet will be :

(Given, $\varepsilon_0 = 8.85 \times 10^{-12}\ \frac{F}{m}$ and acceleration due to gravity, $g = 10\ m/s^2$)

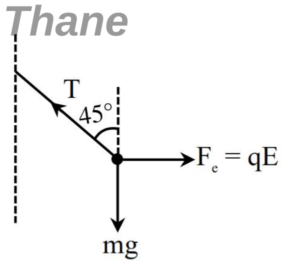


(1) $0.885\ nC/m^2$ (2) $17.7\ nC/m^2$

(3) $885\ nC/m^2$ (4) $1.77\ nC/m^2$

Ans. (4)

Sol.



$qE = mg$

$$q\left[\frac{\sigma}{2\varepsilon_0}\right] = mg$$

$$\sigma = \frac{2\varepsilon_0 mg}{q}$$

$$\sigma = \frac{2 \times 8.85 \times 10^{-12} \times 100 \times 10^{-6} \times 10}{10 \times 10^{-6}}$$

$\sigma = 17.7 \times 10^{-10}\ C/m^2$

$\sigma = 1.77\ nC/m^2$

45. A monochromatic light is incident on a metallic plate having work function ϕ. An electron, emitted normally to the plate from a point A with maximum kinetic energy, enters a constant magnetic field, perpendicular to the initial velocity of electron. The electron passes through a curve and hits back the plate at a point B. The distance between A and B is :

(Given : The magnitude of charge of an electron is e and mass is m, h is Planck's constant and c is velocity of light. Take the magnetic field exists throughout the path of electron)

(1) $\sqrt{2m\left(\frac{hc}{\lambda} - \phi\right)} / eB$ (2) $\sqrt{m\left(\frac{hc}{\lambda} - \phi\right)} / eB$

(3) $\sqrt{8m\left(\frac{hc}{\lambda} - \phi\right)} / eB$ (4) $2\sqrt{m\left(\frac{hc}{\lambda} - \phi\right)} / eB$

Ans. (3)

Sol. $KE_{max} = \frac{hc}{\lambda} - \phi$

$p = \sqrt{2mK_{max}}$

$p = \sqrt{2m\left(\frac{hc}{\lambda} - \phi\right)}$

$d_{A-B} = 2R$

$= 2\left[\frac{p}{qB}\right]$

$$d_{AB} = \frac{2\sqrt{2m\left(\frac{hc}{\lambda} - \phi\right)}}{eB} = \frac{\sqrt{8m\left(\frac{hc}{\lambda} - \phi\right)}}{eB}$$

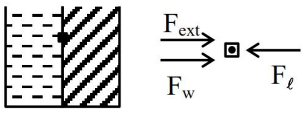
SECTION-B

46. A vessel with square cross–section and height of 6 m is vertically partitioned. A small window of 100 cm^2 with hinged door is fitted at a depth of 3 m in the partition wall. One part of the vessel is filled completely with water and the other side is filled with the liquid having density 1.5×10^3 kg/m^3. What force one needs to apply on the hinged door so that it does not get opened ?

(Acceleration due to gravity = 10 m/s^2)

Ans. **(150)**

Sol.



in equilibrium

$F_{ext} + F_w = F_\ell$

$\Rightarrow F_{ext} = F_\ell - F_w$

$= (P_0 + \rho_\ell gh)A - (P_0 + \rho_w gh)A$

$= (\rho_\ell - \rho_w)ghA$

$= (1500 - 1000) \times 10 \times 3 \times (100 \times 10^{-4})$

$= 150$ m

47. A steel wire of length 2 m and Young's modulus 2.0×10^{11} Nm^{-2} is stretched by a force. If Poisson ratio and transverse strain for the wire are 0.2 and 10^{-3} respectively, then the elastic potential energy density of the wire is ____ $\times 10^5$ (in SI units)

Ans. **(25)**

Sol. $\ell = 2m$; $Y = 2 \times 10^{11}$ $\frac{N}{m^2}$

$$\mu = -\frac{\left(\frac{\Delta r}{r}\right)}{\left(\frac{\Delta\ell}{\ell}\right)} \Rightarrow \frac{\Delta\ell}{\ell} = \frac{1}{\mu} \times \left(\frac{\Delta r}{r}\right)$$

$$= \frac{1}{0.2} \times (10^{-3})$$

$$\Rightarrow \frac{\Delta\ell}{\ell} = 5 \times 10^{-3}$$

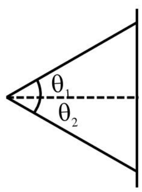
$$u = \frac{1}{2} y\varepsilon_\ell^2 = \frac{1}{2} \times 2 \times 10^{11} \times [5 \times 10^{-3}]^2$$

$= 25$

48. If the measured angular separation between the second minimum to the left of the central maximum and the third minimum to the right of the central maximum is 30° in a single slit diffraction pattern recorded using 628 nm light, then the width of the slit is ____ μm.

Ans. **(6)**

Sol.



$\theta_1 = \sin^{-1}\left(\frac{2\lambda}{a}\right)$

$\theta_2 = \sin^{-1}\left(\frac{3\lambda}{a}\right)$

$\because$ $\theta_1 + \theta_2 = 30°$

$$\Rightarrow \sin^{-1}\left(\frac{2\lambda}{a}\right)+\sin^{-1}\left(\frac{3\lambda}{a}\right)=\frac{\pi}{6}$$

$$\Rightarrow \frac{2\lambda}{a}\sqrt{1-\left(\frac{3\lambda}{a}\right)^2}+\frac{3\lambda}{a}\sqrt{1+\left(\frac{2\lambda}{a}\right)^2}=\sin\frac{\pi}{6}$$

Here $\lambda =$ 628 nm

After solving

$A = 6.07$ μm

Approximate Method :

$\theta = \theta_1 + \theta_2$

$$\Rightarrow \frac{\pi}{6}=\frac{2\lambda}{a}+\frac{3\lambda}{a}$$

$$\Rightarrow \frac{\pi}{6}=\frac{5}{a}(628\text{nm})$$

$\Rightarrow a = 6$ μm

49. γ_A is the specific heat ratio of monoatomic gas A having 3 translational degrees of freedom. γ_B is the specific heat ratio of polyatomic gas B having 3 translational, 3 rotational degrees of freedom and 1 vibrational mode. If $\frac{\gamma_A}{\gamma_B}=\left(1+\frac{1}{n}\right)$, then the value of n is ______.

Ans. **(3)**

Sol. $\frac{\gamma_A}{\gamma_B}=\frac{f_A+2}{f_A}\times\frac{f_B}{f_B+2}$

$$=\frac{3+2}{3}\times\frac{(6+2)}{(6+2)+2}$$

$$=\frac{5}{3}\times\frac{8}{10}=\frac{40}{30}$$

$$\because \frac{40}{30}=1+\frac{1}{n}$$

$$\Rightarrow \frac{40}{30}-1=\frac{1}{n}$$

$\Rightarrow n = 3$

50. A person travelling on a straight line moves with a uniform velocity v_1 for a distance x and with a uniform velocity v_2 for the next $\frac{3}{2}x$ distance. The average velocity in this motion is $\frac{50}{7}$ m/s. If v_1 is 5 m/s then $v_2 =$ _____ m/s.

Ans. **(10)**

Sol. $v_{avg}=\frac{x_1+x_2}{t_1+t_2}$

$$\Rightarrow \frac{50}{7}=\frac{x+\frac{3x}{2}}{\frac{x}{5}+\frac{3x}{2v_2}}$$

$$\Rightarrow \frac{50}{7}=\frac{5/2}{\frac{1}{5}+\frac{3}{2v_2}}$$

$$\Rightarrow \frac{1}{5}+\frac{3}{2v_2}=\frac{7}{20}$$

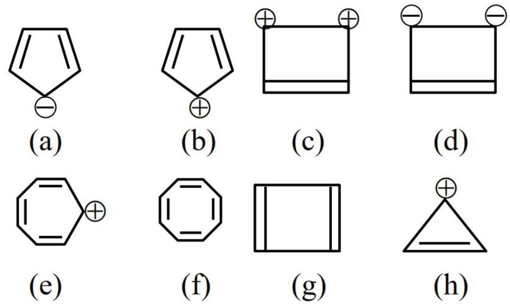
$$\Rightarrow \frac{3}{2v_2}=\frac{7}{20}-\frac{1}{5}=\frac{7-4}{20}$$

$$\Rightarrow \frac{3}{2v_2}=\frac{3}{20}$$

$\Rightarrow v_2 = 10$ m/s

SECTION-A

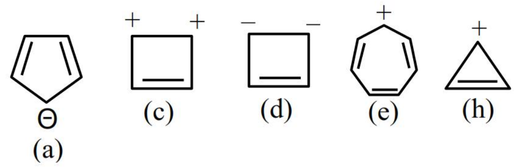
51. Designate whether each of the following compounds is aromatic or not aromatic.



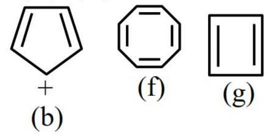
(1) e, g aromatic and a, b, c, d, f, h not aromatic
(2) b, e, f, g aromatic and a, c, d, h not aromatic
(3) a, b, c, d aromatic and e, f, g, h not aromatic
(4) a, c, d, e, h aromatic and b, f, g not aromatic

Ans. **(4)**

Sol. Aromatic compounds



a, c, d, e, h follow Huckel's rule



(Not Aromatic)

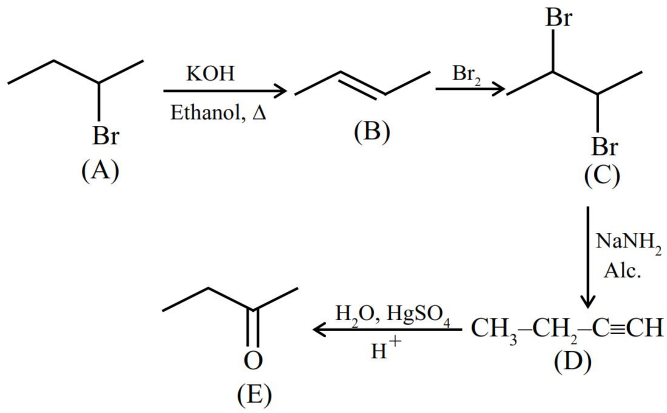
b, f, g, are not aromatic, these compounds do not follow Huckel's rule

52. An optically active alkyl halide C_4H_9Br [A] reacts with hot KOH dissolved in ethanol and forms alkene [B] as major product which reacts with bromine to give dibromide [C]. The compound [C] is converted into a gas [D] upon reacting with alcoholic $NaNH_2$. During hydration 18 gram of water is added to 1 mole of gas [D] on warming with mercuric sulphate and dilute acid at 333 K to form compound [E]. The IUPAC name of compound [E] is :

(1) But-2-yne (2) Butan-2-ol
(3) Butan-2-one (4) Butan-1-al

Ans. **(3)**

Sol.



Butan-2-one

53. The property/properties that show irregularity in first four elements of group-17 is/are :

(A) Covalent radius

(B) Electron affinity

(C) Ionic radius

(D) First ionization energy

Choose the **correct** answer from the options given below:

(1) B and D only (2) A and C only

(3) B only (4) A, B, C and D

Ans. **(3)**

Sol. The order of first four elements of group-17 are as follows.

F < Cl < Br < I (Covalent radius)

Cl > F > Br > I (Electron affinity)

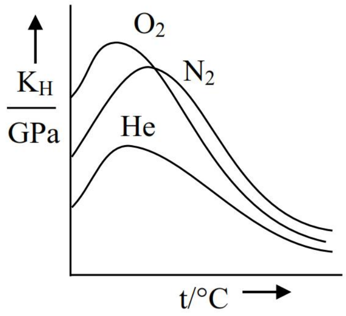
$F^- < Cl^- < Br^- < I^-$ (Ionic radius)

F > Cl > Br > I (I^{st} ionization energy)

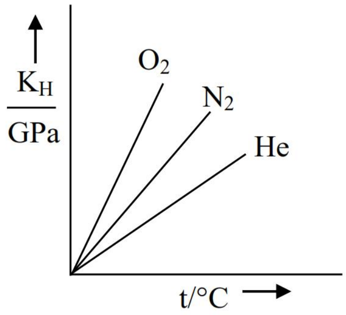
Electron affinity order is irregular.

54. Which of the following graph correctly represents the plots of K_H at 1 bar gases in water versus temperature ?

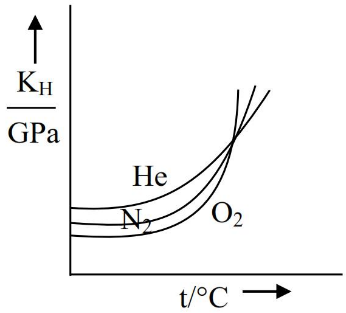
(1)



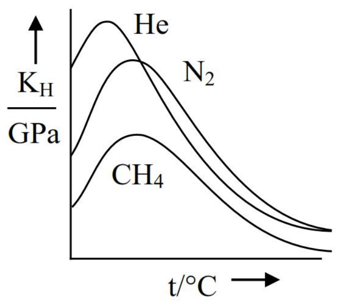
(2)



(3)



(4)



Ans. (4)

Sol. As temperature increases solubility first decrease then increase hence K_H first increase than decrease also at moderate temperature K_H value He > N_2 > CH_4.

55. According to Bohr's model of hydrogen atom, which of the following statement is **incorrect**?

(1) Radius of 3rd orbit is nine times larger than that of 1st orbit.

(2) Radius of 8th orbit is four times larger than that of 4th orbit.

(3) Radius of 6th orbit is three time larger than that of 4th orbit.

(4) Radius of 4th orbit is four times larger than that of 2nd orbit.

Ans. (3)

Sol. $r \propto n^2$

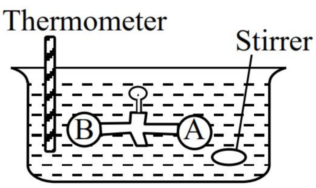
(1) $\frac{r_3}{r_1} = \frac{9}{1}$

(2) $\frac{r_8}{r_4} = \frac{64}{16} = 4$

(3) $\frac{r_6}{r_4} = \left(\frac{6}{4}\right)^2 = \frac{9}{4}$

(4) $\frac{r_4}{r_2} = \left(\frac{4}{2}\right)^2 = 4$

56.



Two vessels A and B are connected via stopcock. The vessel A is filled with a gas at a certain pressure. The entire assembly is immersed in water and is allowed to come to thermal equilibrium with water. After opening the stopcock the gas from vessel A expands into vessel B and no change in temperature is observed in the thermometer. Which of the following statement is **true**?

(1) $dw \neq 0$

(2) $dq \neq 0$

(3) $dU \neq 0$

(4) The pressure in the vessel B before opening the stopcock is zero.

Ans. (4)

Sol. It is free expansion of gas $\Rightarrow P_{ext} = 0$

Where $w = 0$, $q = 0$ and $\Delta U = 0$

57. A solution is made by mixing one mole of volatile liquid A with 3 moles of volatile liquid B. The vapour pressure of pure A is 200 mm Hg and that of the solution is 500 mm Hg. The vapour pressure of pure B and the least volatile component of the solution, respectively, are :

(1) 1400 mm Hg, A (2) 1400 mm Hg, B

(3) 600 mm Hg, B (4) 600 mm Hg, A

Ans. **(4)**

Sol. $P_S = P_A^o.X_A + P_B^o.X_B$

$500 = 200 \times \frac{1}{4} + P_B^o.\frac{3}{4}$

$P_B^o = 600$ mm Hg

As $P_A^o < P_B^o \Rightarrow$ A is least volatile.

58. $CaCO_3(s) + 2HCl(aq) \rightarrow CaCl_2(aq) + CO_2(g)\ H_2O(l)$

Consider the above reaction, what mass of $CaCl_2$ will be formed if 250 mL of 0.76 M HCl reacts with 1000 g of $CaCO_3$?

(Given : Molar mass of Ca, C, O, H and Cl are 40, 12, 16, 1 and 35.5 g mol^{-1}, respectively)

(1) 3.908 g

(2) 2.636 g

(3) 10.545 g

(4) 5.272 g

Ans. **(3)**

Sol. $CaCO_3 + 2HCl \rightarrow CaCl_2 + CO_2 + H_2O$

Moles of $CaCO_3 = \frac{1000}{100} = 10$

Moles of HCl $= 0.76 \times \frac{250}{1000} = 0.19$ (L.R.)

Moles of $CaCl_2$ formed $= \frac{0.19}{2}$

Mass of $CaCl_2 = \frac{0.19}{2} \times 111 = 10.545$ gm

59. If equal volumes of AB_2 and XY (both are salts) aqueous solutions are mixed, which of the following combination will give a precipitate of AY_2 at 300 K?

(Given K_{sp} (at 300 K) for $AY_2 = 5.2 \times 10^{-7}$)

(1) 3.6×10^{-3} M AB_2, 5.0×10^{-4} M XY

(2) 2.0×10^{-4} M AB_2, 0.8×10^{-3} M XY

(3) 2.0×10^{-2} M AB_2, 2.0×10^{-2} M XY

(4) 1.5×10^{-4} M AB_2, 1.5×10^{-3} M XY

Ans. **(3)**

Sol. When equal volumes are mixed molarity reduce to half.

For precipitation $Q_{SP} = [A^{+2}]\,[Y^-]^2 > K_{SP}$

(1) $Q_{SP} = (1.8 \times 10^{-3}) \left(\frac{5}{2}\times10^{-4}\right)^2 < K_{SP}$

(2) $Q_{SP} = (10^{-4})\,(0.4 \times 10^{-3})^2 < K_{SP}$

(3) $Q_{SP} = (10^{-2})\,(10^{-2})^2 > K_{SP}$

(4) $Q_{SP} = \left(\frac{1.5}{2}\times10^{-4}\right)\left(\frac{1.5}{2}\times10^{-3}\right)^2 < K_{SP}$

60. Among SO_2, NF_3, NH_3, XeF_2, ClF_3 and SF_4, the hybridization of the molecule with non-zero dipole moment and highest number of lone-pairs of electrons on the central atom is

(1) sp^3 (2) dsp^2

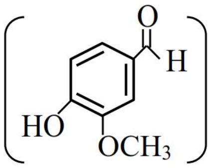
(3) sp^3d^2 (4) sp^3d

Ans. **(4)**

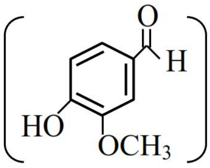
Sol.

Molecule	Hybridisation	Dipole Moment	Lone pair on the central atom
SO_2	sp^2	Non-zero	1
NF_3	sp^3	Non-zero	1
NH_3	sp^3	Non-zero	1
XeF_2	sp^3d	zero	3
$C\ell F_3$	sp^3d	Non-zero	2
SF_4	sp^3d	Non-zero	1

61. Given below are two statements :

Statement (I) : Vanillin 

will react with NaOH and also with Tollen's reagent.

Statement (II) : Vanillin 

will undergo self aldol condensation very easily.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

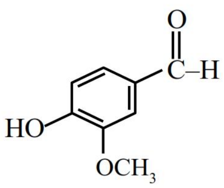
(1) **Statement I** is incorrect but **Statement II** is correct

(2) **Statement I** is correct but **Statement II** is incorrect

(3) Both **Statement I** and **Statement II** are incorrect

(4) Both **Statement I** and **Statement II** are correct

Ans. (2)

Sol. 

Phenolic group soluble in NaOH

Benzaldehyde derivative react with Tollen's reagent.

Vanillin does not give self-aldol reaction due to lack of acidic H for condensation.

62. Identify the correct statement among the following:

(1) All naturally occurring amino acids except glycine contain one chiral centre.

(2) All naturally occurring amino acids are optically active.

(3) Glutamic acid is the only amino acid that contains a –COOH group at the side chain.

(4) Amino acid, cysteine easily undergo dimerization due to the presence of free SH group.

Ans. (4)

Sol. * Isoleucine has 2 chiral centre

* Glycine is optically inactive

* Aspartic acid also contain COOH group at the side chain.

* Cysteine easily dimmerise due to free SH group

63. The correct order of basic nature on aqueous solution for the bases NH_3, $H_2N{-}NH_2$, $CH_3CH_2NH_2$, $(CH_3CH_2)_2NH$ and $(CH_3CH_2)_3N$ is :

(1) $NH_3 < H_2N{-}NH_2 < (CH_3CH_2)_3N < CH_3CH_2NH_2 < (CH_3CH_2)_2NH$

(2) $NH_3 < H_2N{-}NH_2 < CH_3CH_2NH_2 < (CH_3CH_2)_2NH < (CH_3CH_2)_3N$

(3) $H_2N{-}NH_2 < NH_3 < (CH_3CH_2)_3N < CH_3CH_2NH_2 < (CH_3CH_2)_2NH$

(4) $NH_2{-}NH_2 < NH_3 < CH_3CH_2NH_2 < (CH_3CH_2)_3N < (CH_3CH_2)_2NH$

Ans. (4)

Sol. Basic strength of amine depends on hydrogen bonding and electronic inductive effect.

$$NH(Et)_2 > N(Et)_3 > NH_2Et > \ddot{N}H_3 > NH_2 - NH_2$$

64. Given below are two statements :

Statement (I) : The metallic radius of Al is less than that of Ga.

Statement (II) : The ionic radius of Al^{3+} is less than that of Ga^{3+}.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) Both **Statement I** and **Statement II** are incorrect

(2) **Statement I** is incorrect but **Statement II** is correct

(3) **Statement I** is correct but **Statement II** is incorrect

(4) Both **Statement I** and **Statement II** are correct

Ans. (2)

Sol. $\Rightarrow$ The metallic radius order of Al & Ga is

$$B < \underbrace{Ga < Al}_{\downarrow} < In < T\ell$$

(due to poor shielding of d-subshell electrons)

$\Rightarrow$ The ionic radius order of Al^{+3} & Ga^{+3} is

$B^{+3} < Al^{+3} < Ga^{+3} < In^{+3} < T\ell^{+3}$

65. Given below are two statements :

Statement (I) : In octahedral complexes, when $\Delta_o < P$ high spin complexes are formed. When $\Delta_o > P$ low spin complexes are formed.

Statement (II) : In tetrahedral complexes because of $\Delta_t < P$, low spin complexes are rarely formed.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) **Statement I** is correct but **Statement II** is incorrect.

(2) Both **Statement I** and **Statement II** are incorrect

(3) **Statement I** is incorrect but **Statement II** is correct

(4) Both **Statement I** and **Statement II** are correct

Ans. (4)

Sol. In octahedral complex (CN = 6)

If $\Delta_0 <$ P.E. , then high spin complexes are formed

If $\Delta_0 >$ P.E. , then low spin complexes are formed

But in tetrahedral complex (CN = 4)

$\Delta_t <$ P.E. , then mainly high spin complexes are formed and rarely low spin complexes are formed.

66. Choose the correct tests with respective observations.

(A) $CuSO_4$ (acidified with acetic acid) + $K_4[Fe(CN)_6] \rightarrow$ Chocolate brown precipitate.

(B) $FeCl_3 + K_4[Fe(CN)_6] \rightarrow$ Prussian blue precipitate.

(C) $ZnCl_2 + K_4[Fe(CN)_6]$, neutralised with NH_4OH $\rightarrow$ White or bluish white precipitate.

(D) $MgCl_2 + K_4[Fe(CN)_6] \rightarrow$ Blue precipitate.

(E) $BaCl_2 + K_4[Fe(CN)_6]$, neutralised with NaOH $\rightarrow$ White precipitate.

Choose the **correct** answer from the options given below :

(1) A, D and E only (2) B, D and E only

(3) A, B and C only (4) C, D and E only

Ans. (3)

Sol.

$$2CuSO_4 + K_4[Fe(CN)_6] \xrightarrow{CH_3COOH} \underset{\text{(Chocolate brown ppt.)}}{Cu_2[Fe(CN)_6]} + 2K_2SO_4$$

$$4FeCl_3 + 3K_4[Fe(CN)_6] \longrightarrow \underset{\text{(Prussian Blue ppt.)}}{Fe_4[Fe(CN)_6]_3} + 12KCl$$

$$3ZnCl_4 + 2K_4[Fe(CN)_6] \xrightarrow{NH_4OH} \underset{\text{(White or bluish white ppt.)}}{K_2Zn_3[Fe(CN)_6]_2} + 6KCl$$

67. On complete combustion 1.0 g of an organic compound (X) gave 1.46 g of CO_2 and 0.567 g of H_2O. The empirical formula mass of compound (X) is ________ g.

(Given molar mass in g mol^{-1} C : 12, H : 1, O : 16)

(1) 30 (2) 45 (3) 60 (4) 15

Ans. (1)

Sol. Moles of 'C' = $n_{CO_2} = \frac{1.46}{44} = 0.033$

Moles of 'C' = $W_C = 0.033 \times 12$

Moles of 'H' = $2 \times n_{H_2O} = 2 \times \frac{0.567}{18} = 0.063$

Mass of 'H' = 0.0063

Mass of Oxygen (O) = $1 - (W_C + W_H)$

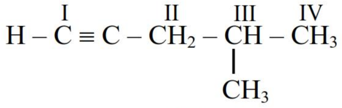
$= 1 - (0.033 \times 12 + 0.063 \times 1) = 0.541$ gm

Moles of 'O' = $\frac{0.541}{16} = 0.033$

Empirical formula = CH_2O

Empirical formula mass = 30.

68. Consider the following compound (X)



(X)

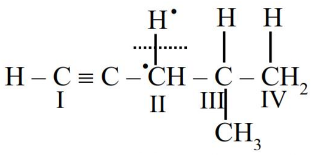
The most stable and least stable carbon radicals, respectively, produced by homolytic cleavage of corresponding C – H bond are :

(1) II, IV (2) III, II

(3) I, IV (4) II, I

Ans. (4)

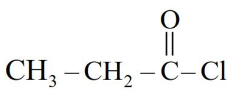
Sol.



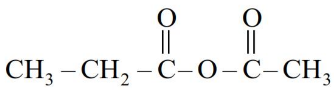
II most stable carbon radical due to resonance stablise

I least stable carbon radical due to no stabilising factor.

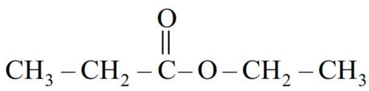
69. Consider the following molecules :



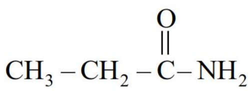
(p)



(q)



(r)



(s)

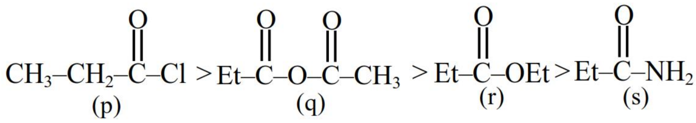
The correct order of rate of hydrolysis is :

(1) r > q > p > s (2) q > p > r > s

(3) p > r > q > s (4) p > q > r > s

Ans. (4)

Sol. Rate of hydrolysis ∝ Leaving group ability



70. A molecule with the formula AX_4Y has all it's elements from p-block. Element A is rarest, monoatomic, non-radioactive from its group and has the lowest ionization enthalpy value among A, X and Y. Elements X and Y have first and second highest electronegativity values respectively among all the known elements. The shape of the molecule is :

(1) Square pyramidal

(2) Octahedral

(3) Pentagonal planar

(4) Trigonal bipyramidal

Ans. (1)

Sol. Given A is rarest, monoatomic, non-radioactive p-block element and form AX_4Y type of molecule.

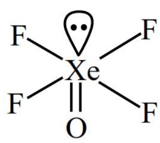
∴ It is concluded that it is Xe

It is given the electronegativity of A is less than X & Y

It is given the electronegativity of X & Y is highest and second highest respectively among all element.

∴ X & Y are F & O

∴ Compound is consider as $XeOF_4$ with square pyramidal shape.



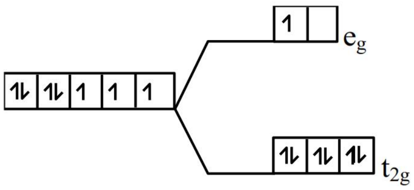
SECTION-B

71. A transition metal (M) among Mn, Cr, Co and Fe has the highest standard electrode potential (M^{3+}/M^{2+}). It forms a metal complex of the type $[M(CN)_6]^{4-}$. The number of electrons present in the e_g orbital of the complex is ________.

Ans. (1)

Sol. Co has highest standard electrode potential (M^{+3}/M^{+2}) among Mn, Cr, Co, Fe

$\therefore$ Complex is $[Co(CN)_6]^{4-}$ and its splitting is as follows.



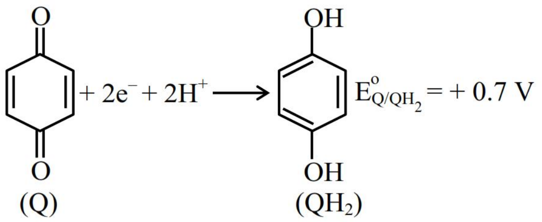
$\therefore$ electron in e_g orbital is one.

72. Consider the following electrochemical cell at standard condition.

$Au(s)|QH_2,Q|NH_4X(0.01M)||Ag^+(1M)|Ag(s)$

$E_{cell} = +0.4V$

The couple QH_2/Q represents quinhydrone electrode, the half cell reaction is given below



$$\left[\text{Given} : E^o_{Ag^+/Ag} = +0.8V \text{ and } \frac{2.303RT}{F} = 0.06V\right]$$

The pK_b value of the ammonium halide salt (NH_4X) used here is ______. (nearest integer)

Ans. (6)

Sol. $QH_2 + 2Ag^+ \rightarrow 2Ag + Q + 2H^+$

$$E = E° - \frac{0.06}{2} \log [H^+]^2$$

$$E = E° - 0.06 \times \log [H^+]$$

$$pH = -\log (H^+) = \frac{E - E^o}{0.06} = \frac{0.4 - 0.1}{0.06}$$

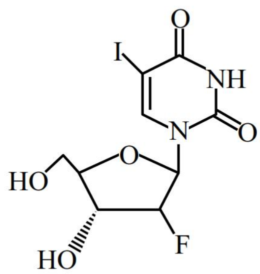
$$= \frac{0.3}{0.06} = 5$$

$$pH + NH_4X = 7 - \frac{1}{2}pK_b - \frac{1}{2}\log C$$

$$5 = 7 - \frac{1}{2} \times pK_b - \frac{1}{2}\log(10^{-2})$$

$pK_b = 6.$

73. 0.1 mol of the following given antiviral compound (P) will weigh _______ $\times 10^{-1}$ g

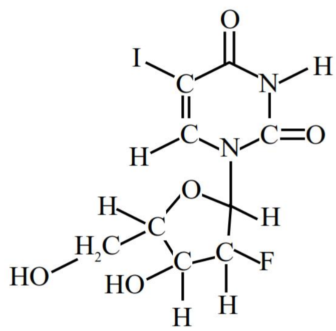


(P)

(Given : molar mass in g mol^{-1} H: 1, C : 12, N : 14, O : 16, F : 19, I : 127)

Ans. (372)

Sol.



Molar mass = 372 gm

$\therefore$ 0.1 mole has = 372×10^{-1} gm

74. Consider the following equilibrium,

$$CO(g) + 2H_2(g) \rightleftharpoons CH_3OH(g)$$

0.1 mol of CO along with a catalyst is present in a 2 dm^3 flask maintained at 500 K. Hydrogen is introduced into the flask until the pressure is 5 bar and 0.04 mol of CH_3OH is formed. The K_p^0 is ________ $\times 10^{-3}$ (nearest integer).

Given : $R = 0.08\ dm^3\ bar\ K^{-1}\ mol^{-1}$

Assume only methanol is formed as the product and the system follows ideal gas behaviour.

Ans. (74)

Sol.

	$CO(g)$	$+\ 2H_2(g)$	$\rightleftharpoons CH_3OH(g)$
$t = 0$	0.1 mol	a mol	–
t_{eq}	$0.1 - x$	$a - 2x$	$x = 0.04$
	$= 0.06$	$= a - 0.08$	
		$= 0.23 - 0.08$	
		$= 0.15$ mole	

$V = 2L$

$T = 500\ K$

$P_{total} = 5$ bar

$$n_{Total} = 0.25 = \frac{1}{4} \text{ mol.}$$

$$P_{total} = n_{total} \times \frac{RT}{V}$$

$$\Rightarrow 5 = (0.06 + a - 0.08 + 0.04) \times \frac{0.08 \times 500}{2}$$

$$\Rightarrow 10 = (0.02 + a) \times 0.08 \times 500$$

$$\Rightarrow a = 0.25 - 0.02 = 0.23 \text{ mol.}$$

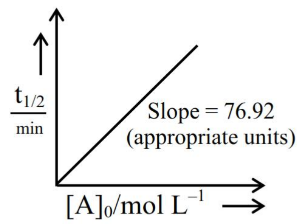
$$K_P = \frac{X_{CH_3OH}}{X_{CO} \times X_{H_2}^2} \times \frac{1}{(P_T)^2} = \frac{0.04}{0.06 \times (0.15)^2} \times \left[\frac{1/4}{5}\right]^2$$

$$= \frac{4}{6 \times (0.15)^2 \times 16} \times \frac{1}{25}$$

$$= \frac{100 \times 100}{24 \times 225 \times 25} = \frac{100 \times 100}{135000}$$

$$= 0.074 = 74 \times 10^{-3}$$

75. For the reaction $A \rightarrow$ products.



The concentration of A at 10 minutes is ________ $\times 10^{-3}$ mol L^{-1} (nearest integer).

The reaction was started with 2.5 mol L^{-1} of A.

Ans. (2435)

Sol. $t_{1/2} \propto [A]_0 \Rightarrow$ Order = zero

$$t_{1/2} = \frac{A_0}{2K} \Rightarrow \text{Slope} = \frac{1}{2K} = 76.92$$

$$K = \frac{1}{2 \times 76.92}$$

$$[A]_{10} = -Kt + A_0 = -\frac{1}{2 \times 76.92} \times 10 + 2.5 = 2.435$$

$$= 2435 \times 10^{-3} \text{ mol/L}$$